**Poisson Process:**

**DEFINITION:** Poisson process is a continuous-time counting stochastic process obtained from a Binomial counting process when its frame size ∆ decreases to 0 while the arrival rate λ remains constant.

Let X (t) = No. of arrivals occurring until time t.

T= inter arrival time

T k= the of k t h arrival

X (t) = Poisson (λ t)

T = Exponential (λ)

T k = Gamma (k, λ)

E X (t) = n p = = λ t

V X (t) = λ t F T (t) = 1-

Probability of k t h arrival before time t

P {T k ≤ t} = P {X (t) ≥k}

P {T k > t} = P {X (t) < k}

**Example 10:**

Customers arrive at a shop at the rate of 2 per minute. Find (i) expected number of customers in a 5 min period (iii) the variance of the number of customers in the same period (iii) the probability that there will be at least one customer.

**Solution:** Here;

Number of hits k = 5000, λ = 5 min-1

Expected time = = = 1000 minutes

Standard deviation (σ) = = 14.14

P {T k < 12 hr.} = P {T k <720}

= P

= P

= P (Z<-19.44)

= 0

**Example 11:**

Customers arrive at a shop at the rate of 2 per minute. Find (i) expected number of customers in a 5 minute period (ii) the variance of the number of customers in the same period (iii) the probability that there will be at least one customer.

**Solution:** Here;

λ = 2

t = 5

(i) E (X) = λ t = 5x2 = 10

(ii) V (X) = λ t = 5x2 = 10

(iii) P {X (5) ≥ 1} = 1 – P {X (5) < 1}

= 1 – P {X (5) = 0}

= 1- e- 10

= 0.999

**Example 12:**

Shipments of paper arrive at a printing shop according to a Poisson process at a rate of 0.5 shipments per day.

(i) Find the probability that the printing shop receives more than two shipments in a day.

(ii) If there are more than four days between shipments, all the paper will be used up and the presses will be idle. What is the probability that this will happen?

**Solution:** Here;

Arrival time; λ = 0.5 per day

X (t) = No. of arrival (shipments) in t days, it is Poisson (0.5 t)

T = Inter-arrival time measured in days, it is Exponential (0.5).

(i) P [X (1) > 2] = 1 – P [X (1) ≤ 2]

= 1 – [P [X (1) = 0] + P [X (1) = 1] + P [X (1) = 2]]

= 1 –

= 1 – e -0.5 [1 + 0.5 + 0.125]

= 1 – 0.6065 x 1.625

= 0.014.

(ii) P [T > 4] = d t

= 0.5

= e -0.5 x 4

= 0.135